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Crack propagation in mixed-mode specimens described via multi-parameter fracture mechanics

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Abstract. The main objective of the paper is to employ a multi-parameter fracture mechanics concept to describe crack propagation through a specimen loaded in mixed-mode. This concept in particular was used because it has been shown that application of the generalized fracture mechanics concept can play a key role for materials with specific fracture behaviour, i.e. when fracture processes occur not only in the very vicinity of the crack tip, but also at larger distances from it. Two mixed-mode (I+II) geometries for the investigation of crack behaviour are presented here. The Williams series expansion is used for crack-tip stress field approximation. Then, considering the higher-order terms of the Williams expansion with regard to maximum tangential stress criterion can provide better estimates of the crack deflection angle. The coefficients of the Williams expansion were determined by means of the over-deterministic method for the purposes of this work. This analysis was performed for each cracked configuration, which is very time-consuming and makes the analysis very extensive. The crack propagation angle obtained by means of the generalized fracture criterion is discussed in detail. It was found that single-parameter fracture mechanics is sufficient when applied close to the crack tip and when mode I of loading prevails, while multi-parameter fracture mechanics can be recommended at larger distances from the crack tip and for configurations where mode II becomes dominant.

1. Introduction

It is very important to study crack behaviour under mixed-mode (I+II) loading because it often occurs in practice. It has been observed that cracks try to propagate in such a manner as to minimize loading mode II, and that they attempt to propagate under pure mode I if possible. Nevertheless, before that the crack is mostly subjected to arbitrary mixed-mode loading. Therefore, it is very reasonable to study the crack path of defects that exist in real structures.

Several approaches can be chosen for the investigation of the fracture behaviour of cracked configurations that arise often because of mechanical or thermal loading or just due to the manipulation of these structural elements. The basic approach is single-parameter linear elastic fracture mechanics, which has been studied extensively [1-4]. Nevertheless, this concept is fully valid only for brittle materials, where the singular crack-tip stress field can be described via the stress intensity factor sufficiently, i.e. where plastic deformation does not occur at all, or only to a very small extent. On the other hand, many materials (such as quasi-brittle or elastic-plastic ones) exist in which fracture processes take place at a greater distance from the crack tip and are more complicated. Consequently, the first task



before a complex fracture analysis can begin is to describe the crack-tip stress field reliably. It has been shown that multi-parameter fracture mechanics can be a suitable method of doing this. Furthermore, the multi-parameter approximation of the crack-tip stress field can be included in the fracture criteria that decide crack stability and/or crack propagation direction.

In this work, the maximum tangential stress criterion is utilized to estimate the crack deflection angle under mixed-mode (I+II) loading. The analysis is performed on two selected cracked configurations and the results obtained are discussed with regard to the choice of the proper critical distance at which the criterion should be applied, as well as with regard to the suitable choice of the number of the initial terms of the Williams expansion to be considered for the tangential stress approximation.

2. Methodology and basic theoretical terms

Basic terms and methodology used within the fracture analysis conducted for this research are presented and explained in the following subsections.

2.1. The Williams power series

The basic idea behind this work consists in the application of the Williams expansion [5] for the reconstruction of a crack-tip stress field. The Williams power series was derived for a homogeneous elastic isotropic cracked body with arbitrary remote loading and has the following form:

$$\sigma_{ij} = \sum_{n=1}^{\infty} A_n \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n, \theta) + \sum_{m=1}^{\infty} B_m \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m, \theta), \text{ where } i, j \in \{x, y\}. \quad (1)$$

Thus, each stress tensor component can be expressed as a function of the polar coordinates (r, θ) of the corresponding point, the known functions f_{ij} , g_{ij} and coefficients A_n , B_m depending on the cracked geometry that must be calculated numerically. The series represents a summation over indexes n and m and is used in its truncated form, i.e. N and M terms are considered for mode I and II, respectively.

2.2. Determination of the coefficients of the Williams expansion

As is stated above, it is necessary to calculate the coefficients of the Williams power series in order to be able to reconstruct the crack-tip stress field. Although more methods exist [6-8], one of them seems to be easier to use than the others because of its low requirements with regard to software and mathematical definitions and that is the over-deterministic method (ODM). It is based on the direct application of the equation for the description of displacement vector components:

$$u_i = \sum_{n=0}^{\infty} A_n r^{\frac{n}{2}} f_i(n, \theta, E, \nu) + \sum_{m=0}^{\infty} B_m r^{\frac{m}{2}} g_i(m, \theta, E, \nu), \text{ where } i \in \{x, y\}. \quad (2)$$

A set of nodes (usually at a radial distance from the crack tip) are selected in the numerical model of the cracked specimen and their displacements under mixed-mode loading are calculated. The obtained values are then used as inputs for equation (2) together with the polar coordinates of the corresponding nodes. Taking into account the elastic properties of the material (Young's modulus E and Poisson's ratio ν), it is then possible to solve a set of $2k$ equations, where k is the number of nodes selected around the crack tip. It has to hold that $2k \geq N + M + 2$ in order to satisfy the condition about the over-determined system of equations. More details and recommendations on using the ODM can be found for instance in [9,10].

2.3. The multi-parameter maximum tangential stress (MPMTS) criterion

When the coefficients of the Williams expansion are known, an arbitrary stress tensor component can be reconstructed in both Cartesian and polar coordinates. When the Maximum Tangential Stress (MTS) criterion is used, the tangential stress $\sigma_{\theta\theta}$ has to be expressed via the Williams power series. According to the theory behind the MTS criterion, the crack propagates in the direction of the maximum tangential stress [3]. When the MPMTS form of the criterion is considered, it is necessary to derivate the Williams power series expression of the tangential stress and fulfil the following equation to find the proper crack deflection angle:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0. \quad (3)$$

The procedure of searching for the maximum of the function of the tangential stress is performed numerically using Wolfram Mathematica software [11].

3. Mixed-mode I+II configurations

As mentioned in the previous sections, two mixed-mode configurations have been chosen for the analysis of the crack deflection angle under various levels of mixed-mode conditions: an Eccentric Asymmetric 4-Point-Bending (EA4PB) specimen and a Semi-Circular Bending (SCB) specimen; see figure 1 for more details about the geometry. Both of the cracked configurations make it quite easy to choose a certain level of mixed-mode conditions by changing one selected parameter: in both cases all the following parameters were kept except one: crack eccentricity e in the EA4PB specimen and crack inclination angle β in the SCB specimen.

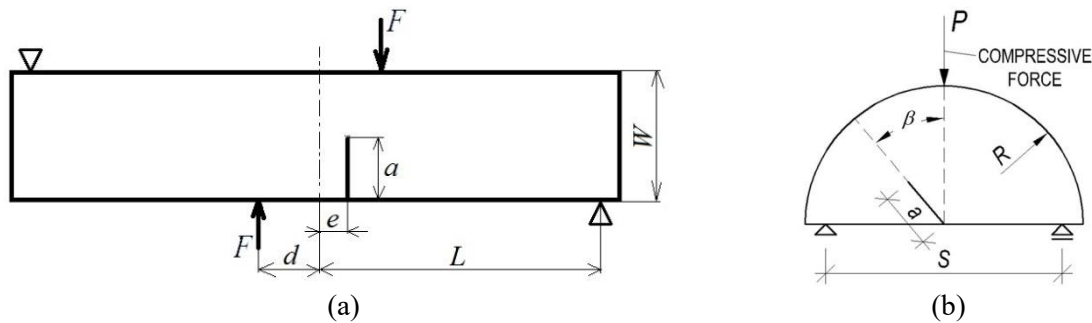


Figure 1. Schema of the mixed-mode I+II cracked configurations under study: (a) Eccentric Asymmetric 4-Point-Bending (EA4PB) specimen; (b) Semi-Circular Bending (SCB) specimen.

The dimensions considered invariable in the numerical models of the specimens presented in figure 1 are as follows (note that the numerical models were modelled as two-dimensional and plane strain conditions were assumed):

- EA4PB:
 - half distance between supports $L = 100$ mm;
 - half distance between loading forces $d = 20$ mm;
 - specimen width $W = 40$ mm;
 - relative crack length $a/W = 0.3$.
- SCB:
 - distance between supports $S = 80$ mm;
 - specimen radius $R = 50$ mm;
 - relative crack length $a/W = 0.3$.

In contrast, the parameters that were varied in order to study various levels of mixed-mode loading were the crack eccentricity in the EA4PB specimen e ($e/W = 0, 0.1, 0.2, 0.3$ and 0.4) and the crack inclination angle β ($\beta = 10, 20, 30, 40$ and 50°), respectively.

The elastic parameters of the material model in the numerical simulations were set according to the properties of common concrete, i.e. Young's modulus 35 GPa and Poisson's ratio 0.2. ANSYS commercial finite element software [12] was used for the simulations and quadrilateral element types (PLANE183) were applied when meshing the cracked specimen with refinement of the mesh at the crack tip.

Note that the MPMTS criterion was applied at three different critical distances ($r_c = 0.1, 0.5$ and 1.0 mm); the influence of this parameter is discussed in the following section.

4. Results and discussion

In the following section, the results obtained based on the multi-parameter linear elastic fracture mechanics analysis are presented. In figure 2, dependences of the crack deflection angle γ estimated by means of the MPMTS criterion on the K_I/K_{II} ratio (which represents the level of the mixed-mode loading) are presented. The dependences were obtained at various critical distances ($r_c = 0.1, 0.5$ and 1.0 mm) at different mixed-mode I+II cracked configurations (EA4PB and SCB) and considering various numbers of the initial Williams expansion terms ($N, M = 1, 2, 3, 4$ and 5).

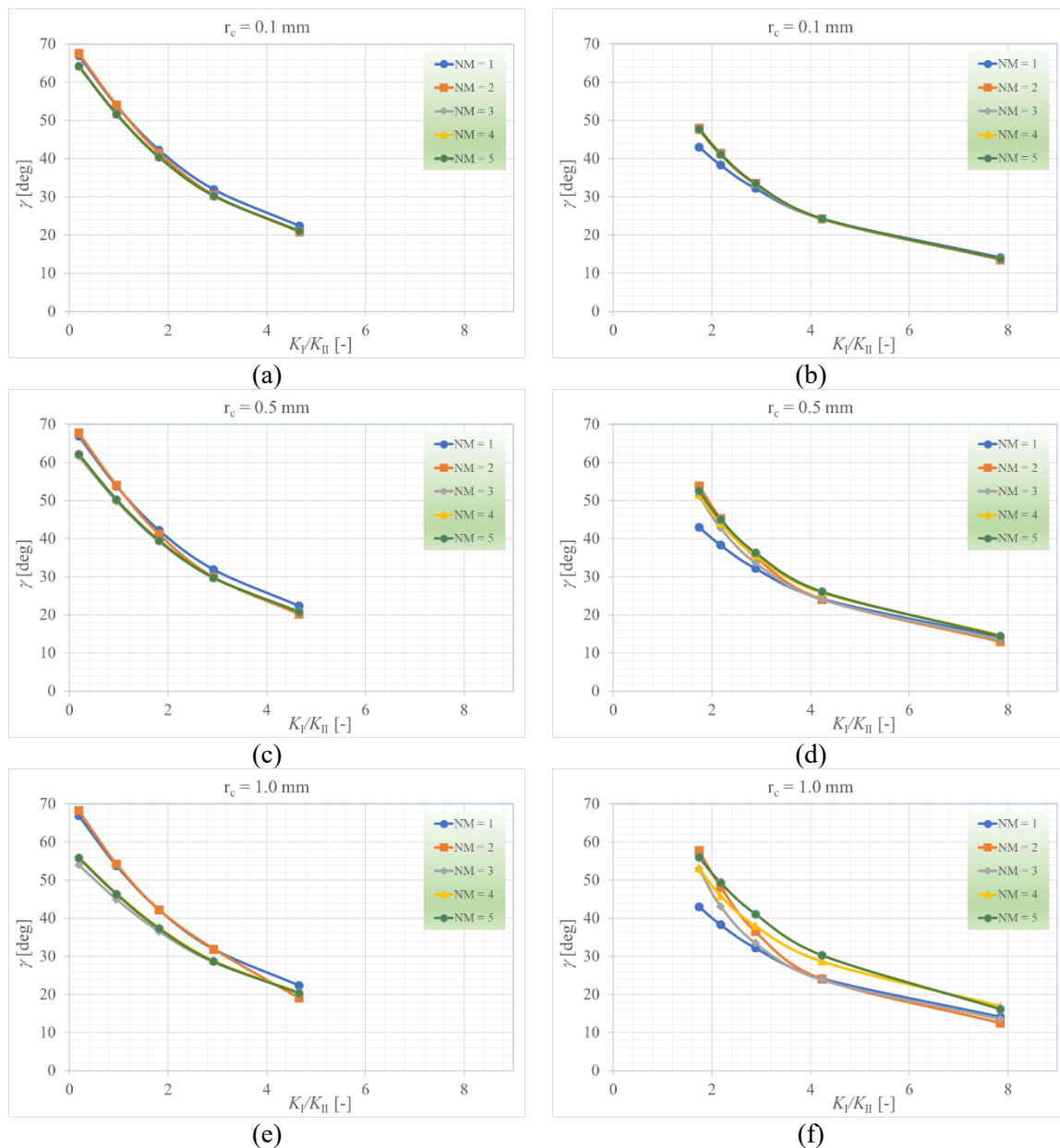


Figure 2. Dependences of the crack deflection angle γ on the mixed-mode level (K_I/K_{II} ratio) obtained by means of the MPMTS criterion at various critical distances from the crack tip taking into account various numbers of the initial Williams expansion terms. (a) EA4PB, (b) SCB, (c) EA4PB, (d) SCB, (e) EA4PB and (f) SCB.

From the dependences presented in figure 2 the following statements can be formulated:

- The mixed-mode I+II conditions modelled via the EA4PB specimen include cases where mode II of loading prevails ($K_{II} > K_I$).
- The resultant crack deflection angles γ obtained for both mixed-mode specimens qualitatively exhibit the same trends.
- At very small distances close to the crack tip ($r_c = 0.1$ mm), the differences between the crack deflection angles γ obtained by considering one or more terms of the Williams expansion are not very significant.
- The differences between the dependences determined by considering various numbers of the Williams expansion terms grow with increasing critical distance r_c at which the MPMTS criterion is applied.
- Similarly, the results of the analysis prove that the higher-order terms of the Williams expansion play a key role when the crack is loaded under mixed-mode I+II conditions with mode II as the dominant mode of loading.

5. Conclusions

Within this work a parametric study was performed on two mixed-mode (I+II) configurations in order to investigate the importance of the higher-order terms of the Williams power expansion used for the approximation of the tangential stress in the multi-parameter maximum tangential stress (MPMTS) criterion. Based on the dependences obtained, it can be concluded that single-parameter fracture mechanics is sufficient when applied close to the crack tip and when mode I of loading prevails, while multi-parameter fracture mechanics can be recommended at larger distances from the crack tip and for configurations where mode II becomes dominant. The choice of the proper distance at which the fracture criterion should be applied can depend on several different factors and has to be made responsibly with regard to material composition and material properties.

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